

# Trigonometry

...a synopsis.

by Dick Furnas

**This picture is great!** It summarizes practically all of the basic relations between the trigonometric functions at once.

• Every right triangle in the picture gives you a trig identity by the Pythagorean theorem:

from the sublime (*and famous*)

$$\square \sin^2 \theta + \cos^2 \theta = 1$$

$$\square 1 + \tan^2 \theta = \sec^2 \theta$$

$$\square 1 + \cot^2 \theta = \csc^2 \theta$$

to the ridiculous (*try this out on your friends in Engineering!*)

$$\csc^2 \theta + \sec^2 \theta = (\tan \theta + \cot \theta)^2$$

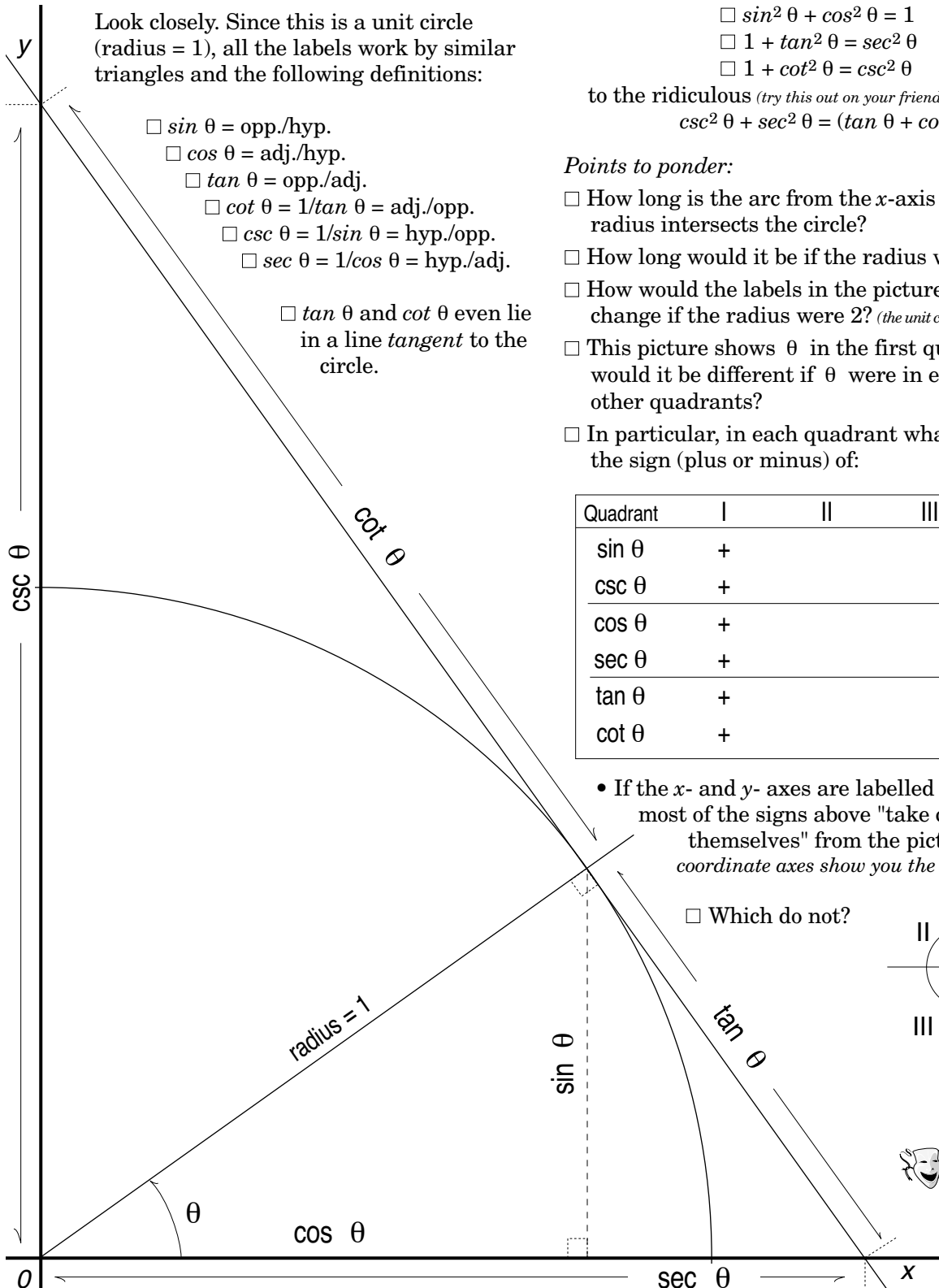
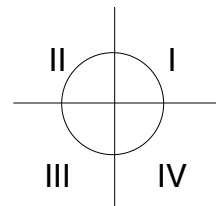
*Points to ponder:*

- How long is the arc from the  $x$ -axis to where the radius intersects the circle?
- How long would it be if the radius were 2?
- How would the labels in the picture have to change if the radius were 2? (*the unit circle is nice, eh?*)
- This picture shows  $\theta$  in the first quadrant. How would it be different if  $\theta$  were in each of the other quadrants?
- In particular, in each quadrant what happens to the sign (plus or minus) of:

| Quadrant      | I | II | III | IV |
|---------------|---|----|-----|----|
| $\sin \theta$ | + |    |     |    |
| $\csc \theta$ | + |    |     |    |
| $\cos \theta$ | + |    |     |    |
| $\sec \theta$ | + |    |     |    |
| $\tan \theta$ | + |    |     |    |
| $\cot \theta$ | + |    |     |    |

• If the  $x$ - and  $y$ - axes are labelled as usual, even most of the signs above "take care of themselves" from the picture. (*i.e. The coordinate axes show you the sign directly.*)

Which do not?



Look closely. Since this is a unit circle (radius = 1), all the labels work by similar triangles and the following definitions:

- $\sin \theta = \text{opp./hyp.}$
- $\cos \theta = \text{adj./hyp.}$
- $\tan \theta = \text{opp./adj.}$
- $\cot \theta = 1/\tan \theta = \text{adj./opp.}$
- $\csc \theta = 1/\sin \theta = \text{hyp./opp.}$
- $\sec \theta = 1/\cos \theta = \text{hyp./adj.}$
- $\tan \theta$  and  $\cot \theta$  even lie in a line *tangent* to the circle.